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SENSITIVITY ANALYSIS FOR OPTIMIZING LARGE SOLAR THERMAL SYSTEMS

Michael Krause, Klaus Vajen, Frank Wiese
Universität Kassel, Institut für Thermische Energietechnik, Kurt-Wolters-Str. 3, D-34109 Kassel, Germany
Phone: +49-561/804-3892, Fax: +49-561/804-3993, e-mail: solar@uni-kassel.de

Hans Ackermann
Universität Marburg, Fachbereich Physik, Renthof 5, D-35032 Marburg, Germany

Abstract – Two techniques have been applied for the determination of sensitivities of the performance of a large solar thermal system on its dimensioning and operation parameters: Differential Sensitivity Analysis (DSA) and Monte-Carlo Analysis (MCA). Since the sensitivities have to be determined for an optimal designed system, automatic optimizations with classical and evolutionary algorithms have been carried out. For this, a TRNSYS system model of the solar thermal system has been implemented and coupled with the optimization algorithms. However, the determination of the sensitivities depends strongly on the chosen range of parameter variations, which is different for each individual parameter. Furthermore, since the influence of the parameters on the system performance is not linear and correlation between parameters exist, the results from DSA has to be interpreted carefully. Especially a comparison of sensitivities of different parameters is difficult. However, the application of MCA for a detection of sensitivities in a minimum of the objective function leads to difficulties concerning the distribution of parameter variation and resulting objective function values, too. Only by using quantiles, a determination of a overall sensitivity is possible depending on the parameter variations carried out.

1. INTRODUCTION

A good system design and proper dimensions of all its components are a prerequisite to assure a good performance of a solar thermal system. Suitable design and dimensions depend on the location of the system, especially climate, the amount and profile of domestic hot water consumption and other boundary conditions such as size and orientation of the roof for the solar collectors. Additionally, also the intentions of the customer, e.g. maximum investment or solar fraction, have to be considered. Taking all these requirements into account, a large number of parameters have to be determined during the planning process of these systems. For this, automatic optimizations using computer simulation programs like TRNSYS (Klein et al., 1994), an established simulation program for solar thermal systems, could be one possible solution. For this, the thermal simulations can be coupled with optimization algorithms. Classical algorithms like gradient methods or the Simplex algorithm as well as evolutionary algorithms like Evolution Strategy and Genetic algorithms can be used. (Krause et al., 2002) has shown that, for the use during the planning process, evolutionary algorithms are more appropriate than classical algorithms. The latter achieve only comparable results if the range of values of the parameters is restricted or if the number of parameters is small, as it is the case for optimizations during the operation of solar thermal systems.

However, even for evolutionary algorithms it is important to reduce the number of parameters for the optimization in order to increase the convergence speed and reach reliable results. Thus, sensitivity analysis can help to detect the most influential parameters.

Different methods exist to determine sensitivities of parameters on an objective function. Most common among these are Differential Sensitivity Analysis (DSA) and Monte-Carlo Analysis (MCA), other techniques are e.g. Factorial Sensitivity Analysis (FSA) and Stochastic Sensitivity Analysis (SSA) (Fürbringer and Roulet, 1995, Lomas and Eppel, 1992).

Besides the knowledge about the importance of the individual parameters, knowledge about the sensitivity itself in the vicinity of the optimum parameter set is necessary. Especially for the planning of solar hot water systems, it has to be clear how accurate the parameters have to be determined and adjusted, to avoid e.g. an increase of the solar heat costs of the system.

2. INVESTIGATED SYSTEM

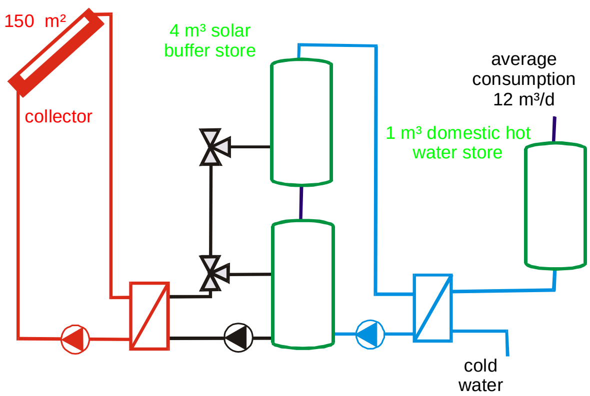

Figure 1: Design of the SHW-system ($f_{sol} \approx 28\%$).

Figure 1 shows the design of the investigated solar hot water (SHW) system installed at a hospital in Frankfurt (Germany). The discharging of the buffer stores of the system takes place only when hot water is taped. This leads to difficulties of the regulation on the primary side of the discharging circuit, and volume dependent UA-values of the heat exchanger have to be considered. The flow rates on both sides of the heat exchanger for the solar charging of the buffer stores are constant.

The whole system has been implemented in TRNSYS, weather data have been generated with Meteonorm (Meteotest et al., 1997) and the input data for the hot water consumption were taken and extrapolated from measurements at another SHW system which is also installed at a hospital.

3. Optimization results

With implementing cost functions of each component the investment of different system configurations can be calculated and hence, with an assumed lifetime of the system of 20 years and an interest rate of 6 % also the annuity. To determine the expected solar gain of the system for the different configurations, one year thermal simulations have been carried out. To consider the electricity demand for pumping (and to select reliable pump sizes), a hydraulic modelling of the collector circuit has been made additionally. Thus, the solar heat cost as expressed by Equation (1) has been used to define the resulting objective function ζ for the optimizations:

$$\zeta = \text{solar heat cost} = \frac{\text{annuity}}{\text{annual solar heat delivery to the hot water storage}} \tag{1}$$

Three different algorithms have been used for the optimization of the system described in section 2: Evolution Strategy, Genetic Algorithm and the algorithm of Simplex. 17 parameters have been varied, including control parameters, flow rates, buffer storage volume, pipe diameters, collector orientation, UA-values of the heat-exchangers and sensor and inlet positions at the buffer store. The size of the collector field has been fixed in order to avoid the use of penalty functions to meet the desired investment of the real system.

Figure 2 shows the development of the objective function against the number of simulations carried out during the optimizations. It can be seen in the diagram that Evolution Strategy and Genetic Algorithm show similar performance if for Genetic Algorithm redundant simulations are neglected. The Simplex algorithm shows very poor performance, but as mentioned in (Krause et al., 2002), the performance of the classical algorithms can be improved if the range of parameter values would be restricted or the number of parameters reduced. Compared to the conventionally planned and installed system, the solar heat cost can be reduced by 16 %. The improvement is mainly caused by a reduction of the pipe diameter in the solar circuit, a higher UA-value of the heat exchanger for the charging of the buffer store and by the reduction of the flow rate in the solar circuit.

However, it is not clear if all cost functions are identically to those assumed during the original planning process and if all boundary conditions are considered in the same way. Thus, the real potential of carrying out optimizations during the planning process might be different.

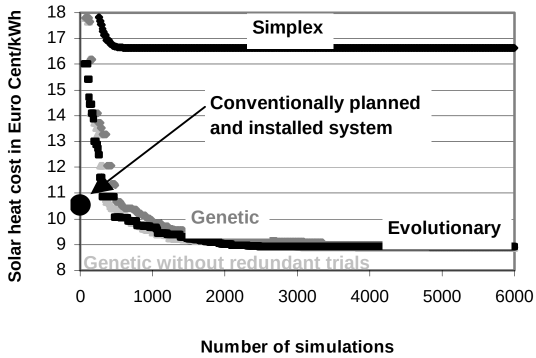

Figure 2: Development of the simulated solar heat cost for the investigated system depending on the number of required simulations. The optimizations have been carried out for three different algorithms.

4. SENSITIVITY ANALYSIS

4.1 Techniques for sensitivity analysis

In this work, two different techniques for sensitivity analysis have been implemented, Differential Sensitivity Analysis (DSA) and Monte-Carlo Sensitivity Analysis (MSA). Both techniques are investigating the sensitivity of an objective function $f(p)$ on parameters in the vicinity of one special point p_0, whereby the n parameters are represented by the n-dimensional vector p.

The main difference between DSA and MCA is that DSA offers sensitivity information on each parameter independently from the other parameters, whereas with MSA information is only available globally and the sensitivities of the individual parameters can not be derived. However, DSA is only reliable, if the influence of the individual parameters on the objective function are linear over the interesting range and the effects of the parameters are superposable.

With interpreting the δ_j as standard deviations of the individual parameters, summarised in the vector δ, DSA is evaluating 4 values of the objective function for each parameter:

$$f(p_0) - f(p_0 + m \cdot \delta_j \cdot e_j) \quad \text{for } j = 1..n, \quad m \in \{-2, -1, 1, 2\} \tag{2}$$

The test on linearity for each parameter can be performed by comparing both sides of equation (3).

$$f(p_0 + m \cdot \delta_j \cdot e_j) \overset{!}{=} f(p_0) + m[f(p_0 + \delta_j \cdot e_j) - f(p_0)] \quad (3)$$
$$\text{for } j = 1..n, \quad m \in \{-2, -1, 1, 2\}$$

In order to test if the parameters are superposable, equation (4) has to be evaluated:

$$f(p_0 + m \cdot \delta) \overset{!}{=} f(p_0) + \sum_{j=1}^{n} [f(p_0 + m \cdot \delta_j \cdot e_j) - f(p_0)] \quad (4)$$
$$\text{for} \quad m \in \{-2, -1, 1, 2\}$$

If equations (3) and (4) are not fulfilled, it is not possible to detect individual sensitivities with DSA using equation (2).

In opposite to this, MSA is even applicable if equations (3) and (4) are not fulfilled. However, with MSA it is only possible to detect total but not individual sensitivities. For this, to each parameter has to be assigned a probability distribution. If a normal distribution is used, parameter values in the vicinity of p_0 occur more often than values with a higher distance. With a set of N parameter vectors, N evaluations of the objective function have to be carried out. For this, each parameter has to be varied independently and simultaneously referring to their specific distributions. Thus, the total sensitivities can be expressed by the standard deviation defined in equation (5):

$$s = \sqrt{\frac{1}{N-1} \cdot \left(\sum_{n=1}^{N} f(p_n)^2 - N \cdot f(p_0)^2 \right)} \quad (5)$$

s can be estimated after any number of simulations, the accuracy of the estimations can be determined with the calculation of a confidential interval using the χ^2-distribution.

4.2 Investigations with DSA

In the case that the influences of all parameters on an objective function value are not linear and/or dependent from each other, it is necessary to mention the certain point at which a sensitivity is determined, too. Since it is very important for the construction of a SHW system to know how certain the parameters have to be adjusted to guarantee optimal performance, the optimal point of the optimizations from chapter 3 is the main point of interest.

However, in the vicinity of the optimum, the influence of all the parameters cannot be linear, since every variation from the optimal parameter values has to lead to a degradation. Thus, together with the expectation that correlations exist between some parameters, the results from DSA has to be interpreted carefully.

Figure 3 and 4 show exemplary results of the DSA for the influence of the pipe diameter in the collector circuit and the buffer store volume on the solar heat cost and on the solar gain of the system. The grey ranges in the diagrams represent the range where a variation of the parameter leads to a deviation concerning solar heat cost and solar gain respectively of less than 1 %. In Figure 3 it can be seen that the curve of solar heat cost is not symmetrically around the optimum value. Furthermore, the progression is not linear, but could be approximated possibly on both sides by quadratic functions. However, the influence of the pipe diameter on the solar heat cost is quite low in the vicinity of the optimum, only farther parameter values lead to a remarkable increase of solar heat cost, especially for low values. The influence on the solar gain is very low over the whole investigated parameter range.

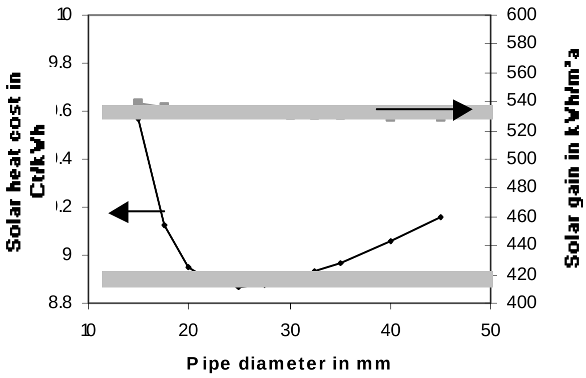

Figure 3: Sensitivity of solar heat cost and solar gain on the pipe diameter, determined with DSA. The grey ranges represent deviations of the objective function and the solar gain respectively of less than 1 %.

The buffer store system is assumed to be composed by some small equally sized stores that are available on the market. Figure 4 shows that the influence of the buffer store volume leads to a non-monotonous progression of the curve, which is caused by different arrangements of the single buffer stores to build the complete buffer storage. Thus, the optimization process is more difficult, since local minima exist in the parameter space.

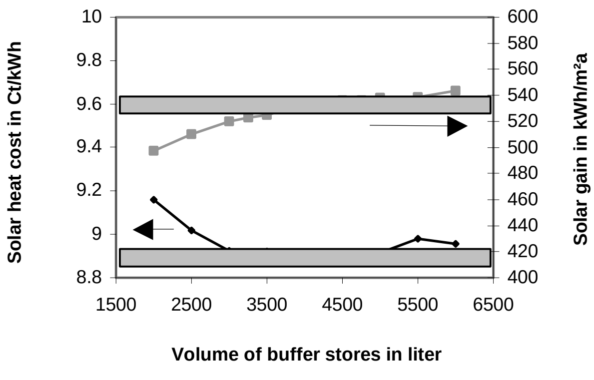

Figure 4: Sensitivity of solar heat cost and solar gain on the buffer store volume, determined with DSA. Again, the grey ranges represent deviations of the objective function and the solar gain respectively of less than 1 %.

For the test if the influences of the parameters are superposable, both sides of equation (4) has been evaluated. With the assumed δ_j, the deviations for $m \in \{-2,-1,1,2\}$ are less than 1.5 %, which signifies that the correlation between the parameters are small in the vicinity of the optimum.

DSA determines the partial derivatives of the function with the help of the difference quotients as expressed in equation (6):

$$D_j f(p) = \frac{f(p_0) - f(p_0 + m \cdot \delta_j \cdot e_j)}{m \cdot \delta_j} \quad j = 1..n \qquad (6)$$

However, finding the most sensitive among the 17 investigated parameters, a criterion has to be found to compare the results of the partial derivatives, and thus the sensitivity of the different parameters. Hereby, difficulties occur due to different ranges of parameter variations of the individual parameters. One possibility would be to use normalized parameters, as defined in equation (7). This would lead to the effect that the sensitivity depends on the determination of minimum and maximum values allowed during the optimization:

$$\overline{p}_j = \frac{p_j}{p_{j,max} - p_{j,min}} \quad \textit{for } j = 1..n \qquad (7)$$

The aim of the optimization is to determine a system configuration which results in an optimal system performance regarding the objective function. However, not every parameter can be adjusted in the real system to the exact value, determined by the optimization. Thus, restrictions of components made by manufacturers or adjustment limits can be used to define a parameter resolution, to which the sensitivity can be related. E.g., buffer stores for large solar thermal systems are usually available in steps of about 500 liter and sensor positions can be fixed in practice for instance with a certainty of 0.05 m.

A vice versa argumentation would be to determine the parameter deviation from the optimum value, until a defined degradation of the objective function value is obtained. However, it still has to be decided whether this deviation means that the parameter is important or not.

Thus, the sensitivity is mainly dependent on the user specified conditions and not easily transferable to other applications.

4.3 Investigations with MCA

Since the influence of a parameter variation is not linear on the objective function and the effects of the individual parameters are not superposable, the results of DSA have to be interpreted carefully. Thus, an investigation with MCA has to be performed additionally.

In order to carry out MCA, a normal distribution of parameter variations has to be generated. Since upper and lower boundaries for some parameters are necessary to run TRNSYS system simulations, it is not possible to generate perfect normal distributions of the variations. Furthermore, it is not possible to obtain a normal distribution from the output values of the objective function by MCA, since no values below the optimal objective function value (solar heat cost) are possible. Thus, the distribution will not be symmetrically.

In addition, the need to determine standard deviations for the single parameters, similar weighting problems occur like for DSA. According to this, also the results of MCA have to be interpreted carefully.

For the MCA investigations, 3000 simulations have been carried out. The standard deviations of the parameter distributions are the same as used for DSA. According to the results of DSA symmetrical quadratic approximations have been determined for the solar heat cost dependent on variations of single parameters. The grey area in Figure 5 shows the normalized frequency of MCA. Additionally, instead of carrying out simulations, the solar heat costs have been calculated using the quadratic approximations for all parameter configurations selected by MCA. The normalized frequency of the approximated solar heat cost is shown in the diagram, too. With this approximation, the resulting frequency distribution has to be similar to a χ^2 - distribution.

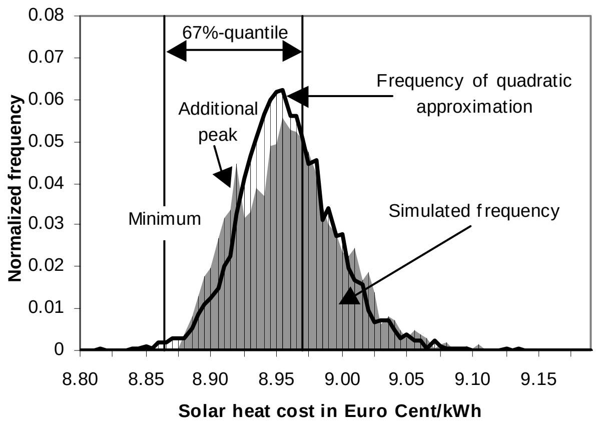

Figure 5: Normalized frequency of the solar heat cost of the different system configurations obtained by 3000 simulations generated with MCA. The grey distribution represents the simulated values, the distribution with the black curve shows the same for quadratic approximations, cf. text.

Figure 5 shows that simulated and approximated distributions are similar, except of an additional peak of the simulated results. This peak indicates that the influences of some parameters are correlated and even if the parameter variations were small due to the use of small standard deviations, quadratic approximations lead to different solar heat costs.

Since the distributions can not be centred around the minimum, equation (5) can not be used to calculate a standard deviation of the distribution of the solar heat cost. However, the 67%-quantile can be calculated. This indicates that 67% of the tested parameter configurations and the assumed standard deviations of the individual parameter distributions lead to a degradation of the solar heat cost of less than 1.2 %.

However, if bigger deviations of the individual parameters are allowed, the shape of the normalized frequency distribution differs more and more from a χ^2-distribution and the 67%-quantile becomes larger, too.

5. CONCLUSIONS

Two different techniques have been investigated to determine the sensitivity of the solar heat cost of large solar thermal systems (SHW) on its system and component parameters: Differential Sensitivity Analysis (DSA) and Monte-Carlo Analysis (MCA). The knowledge of the sensitivity can help to reduce the number of parameters which have to be determined in a planning process. Furthermore, it indicates, how certain each parameter has to be adjusted to guarantee an optimal performance of the system.

Automatic optimizations have been carried out with the help of TRNSYS-simulations to find an optimal system design regarding assumed climate condition and hot water consumption. Even if both classical and evolutionary algorithms have been used, evolutionary algorithms are more suitable for the optimization of solar system in the planning process. Compared to the installed system at a hospital in Frankfurt (Germany), the simulation indicate a possible decrease of the solar heat costs by 16 %. This reduction is mainly caused by a reduction of the pipe diameter in the solar circuit, a higher UA-value of the heat exchanger for the charging of the buffer store and by reducing the flow rate in the solar circuit.

To apply the DSA and MCA techniques, variation ranges have to be defined for each parameter to carry out the analysis. Since the detected sensitivities depend strongly on those definitions, reasonable criteria for their determination have to be found. These can be for instance the resolutions of the sizes of available components. However, due to different weightings of the parameter like buffer store volume and flow rates, the sensitivities on the individual parameters can not be compared in a general way.

Since the influence of the parameters on the objective function is not linear and because of correlation between parameters, the results gained from DSA have to be interpreted carefully. Thus, MCA has to be carried out additionally, even if no individual sensitivities can be detected with MCA. Furthermore, since every parameter variation leads to a degradation of the objective function, MCA can not be interpreted in the usual way. Only the determination of quantiles can help to gain reliable results. However, the results from MCA depend strongly on the defined parameter variation, too.

Thus, general criteria have to be found for the comparison of sensitivities of different parameters. Furthermore, different solar thermal system designs have to be investigated, to determine how the sensitivities depend e.g. on system design, climate conditions and hot water consumption. A comparison between different systems might be independent on the range of parameter variation

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