INVESTIGATION AND COMPARISON OF DIFFERENT MODEL EQUATIONS FOR UNGLAZED COLLECTORS

Klaus Vajen
Elimar Frank
Christopher Jipp
Universität Kassel
Institut für Thermische Energietechnik
34109 Kassel (Germany)
solar@uni-kassel.de

Abstract

Different model equations have been proposed in literature for the description of the thermal behaviour of uncovered collectors. Most of the models are derived from the energy balance at the absorber surface. In the research study presented in this paper, the impact of different approaches to derive a single one-node model-equation on the calculated thermal performance has been investigated. Physically reasonable heat-transfer coefficients the impacts have been investigated of different approaches to derive a single one-node model-equation as well as model equations given in technical norms have been applied. The equation given in (EN 12975, 2002) was shown to be less suitable for the modelling of uncovered collectors, whereas equations suggested in (Perers 1987) and (Vajen et al. 2003) deliver more accurate results for varying boundary conditions. Further investigations are necessary concerning the sensitivity of different parameters on the prediction of the useful solar energy gains.

Introduction

To describe the thermal behaviour of uncovered solar collectors different numerical models can be used. They should be able to deliver the correct thermal power gained by the collector for given measured or generated boundary conditions like weather, fluid inlet temperature and mass flow rate if the parameters of the model have been determined theoretically or experimentally.

Starting point of most of the models is the energy balance of the absorber surface. Usually, the physical formulation in eq. (1) is used for the derivation of a model equation with one thermal node:

$$\dot{q}_{use} + \dot{q}_b + c_{eff} \cdot \frac{\partial T_f}{\partial t} = h_i \cdot \left(T_p - T_f\right) = \dot{q}_{abs} - \dot{q}_{sky} - \dot{q}_{conv} - \dot{q}_{cond} \qquad (1)$$

with

$$\dot{q}_{use} = \dot{v} \cdot c_p \cdot \rho \cdot \left(T_o - T_i\right) \qquad (2)$$

$$T_f = \left(T_o + T_i\right)/2 \qquad (3)$$

$$\dot{q}_{abs} = \alpha \cdot G \qquad (4)$$

$$\dot{q}_{sky} = \varepsilon_{eff} \cdot \sigma \cdot (T_p^4 - T_{sky}^4) \qquad (5)$$

Here, $\dot{q}_{abs}$ is described with a simple relation, but eq. (4) could easily be extended by an incidence angle modifier. Following a common approach, the convective heat transfer is assumed to increase linearly with the wind speed:

$$\dot{q}_{conv} = U_c \cdot \left(T_p - T_a\right) = \left(U_{c0} + U_{c1} \cdot v_W\right) \cdot \left(T_p - T_a\right) \quad (6)$$

$$\dot{q}_b = U_b \left(T_f - T_a\right) \quad (7)$$

describes the heat transfer on the bottom of the collector. Since the average surface temperatures of the back of the absorber and of the underlying roof are difficult to measure, these temperatures are often approximated by T_f and T_a, respectively. Further model assumptions are:

1) The flow through all absorber channels connected in parallel is evenly distributed.
2) Heat transport in flow direction takes place only due to the fluid flow and not e.g. by conduction inside the absorber material.
3) The difference of the upper absorber surface temperature and the corresponding fluid temperature is assumed to be independent of the position in fluid flow direction. This is equivalent to a linear increase of the temperatures of fluid and absorber surface. Thus, the validity of the models is limited to sufficiently high specific flow rates.
4) All heat capacities are lumped together in a single node, represented by the average fluid temperature.
5) The heat transfer is modelled quasi-stationary and the heat transfer coefficients are assumed to be constant for the specific absorber ranges and the time intervals taken into account.
6) Furthermore, the model covers the standard operation temperatures and does not include e.g. heat gains due to condensation. Implicitly a specific arrangement of the temperatures is assumed:

$$T_d \le T_a \le T_i \le T_o \quad (8)$$

With the model described in eqs. (1-7), the collector outlet temperature T_o can not be predicted directly, even if all boundary conditions (G, dV/dt, T_i, T_{sky}) and parameters (c_p, ρ, α, ε, h_i, U's, c_{eff}) are known, because the average absorber surface temperature T_p is very difficult to measure and thus usually unknown. To determine T_p, an iterative algorithm has to be applied: T_p has to be changed until both sides of eq. (1) are fulfilled.
On the other hand, several one-node models for uncovered collectors are described in literature (e.g. (Perers 1987, Vajen et al 1999, EN 12795, 2002, Vajen et. al 2003)), each consisting of a single equation which can be evaluated directly without any iterations. Besides the (ISO 9806, 1995) and (Rockendorf et al. 2001) these models are based on the energy balance of the absorber surface as described in eq. (1). To end up with a single model equation further assumptions are made, especially regarding a linearity of the radiant heat transfer between absorber surface and the surroundings. The model errors due to different assumptions are investigated in the following sections, taking into account varying boundary conditions. The iterative evaluation of eq. (1) uses the least assumptions and can reproduce the thermal behaviour of the collector most precisely. Thus, in the following the model is taken as a reference for the comparison of the accuracy of the different approaches.

Derivation of the single model-equations

Alternative to the iterative calculation the average surface temperature can be approximated by the average fluid temperature plus a constant temperature difference:

$$T_p = T_f + C \quad (9)$$

In this way, the energy balance can be calculated directly. Under real conditions, the temperature difference between T_p and T_f varies.

As already mentioned, it is difficult to measure the absorber plate temperature, whereas the fluid temperature is known in most cases. Thus, the terms including T_p in the second part of eq. (1) are extended to temperature differences (T_p-T_f) which can be eliminated later.
E.g., the convective heat transfer term, of. eq. (6), can be extended as follows:

$$\dot{q}_{conv} = \left(U_{c0} + U_{c1} \cdot v_w\right) \cdot \left[\left(T_p - T_f\right) + \left(T_f - T_a\right)\right] \quad (10)$$

The main differences between the simplified models are contained in the modelling of the radiant heat exchange between absorber surface and the sky. Here, the cubic part of the temperature difference is linearized and approximated by a model parameter. The temperature differences can be expressed in different ways.
In (EN 12795, 2002), eq. (5) is substituted by the following equation:

$$\begin{aligned} \dot{q}_{sky} &= \varepsilon_{eff} \cdot \sigma \cdot \left[\left(T_p^4 - T_f^4\right) + \left(T_f^4 - T_a^4\right) + \left(T_a^4 - T_{sky}^4\right)\right] \\ &= \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_a\right) \cdot \left(T_p^2 + T_a^2\right) \cdot \left[\left(T_p - T_f\right) + \left(T_f - T_a\right)\right] \\ &+ \varepsilon_{eff} \cdot \sigma \cdot \left(T_a^4 - T_{sky}^4\right) \end{aligned} \quad (11)$$

ε_{eff} is an effective emission-coefficient that includes the properties of the absorber-material and the geometric structure of the absorber surface.
In eq. (10), T_p-T_f can be separated and eliminated afterwards. However, T_p^2 still remains in eq.(11). Therefore, for a defined operation point with constant values of T_p and T_a, a heat transfer parameter has to be defined which is assumed to be constant:

$$U_{pa} := \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_a\right) \cdot \left(T_p^2 + T_a^2\right) \quad (12)$$

Hence:

$$\dot{q}_{sky} = U_{pa} \cdot \left[\left(T_p - T_f \right) + \left(T_f - T_a \right) \right] + \varepsilon_{eff} \cdot \sigma \cdot \left(T_a^4 - T_{sky}^4 \right) \qquad (13)$$

(Vajen et al. 2003) extended eq. (11) only by T_f instead of T_f and T_a which leads to

$$\dot{q}_{sky} = \varepsilon_{eff} \cdot \sigma \cdot \left[\left(T_p + T_f \right) \cdot \left(T_p^2 + T_f^2 \right) \cdot \left(T_p - T_f \right) \right] + \varepsilon_{eff} \cdot \sigma \cdot \left(T_f^4 - T_{sky}^4 \right) \qquad (14)$$

with a heat transfer parameter defined as

$$U_{pf} := \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_f \right) \cdot \left(T_p^2 + T_f^2 \right) \qquad (15)$$

and finally

$$\dot{q}_{sky} = U_{pf} \cdot \left(T_p - T_f \right) + \varepsilon_{eff} \cdot \sigma \cdot \left(T_f^4 - T_{sky}^4 \right) \qquad (16)$$

For a direct linearization of eq. (10) in T_p-T_{sky}, as it was carried out by (Perers 1987) and (Vajen et al 1999), the extension delivers

$$\dot{q}_{sky} = \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_{sky} \right) \cdot \left(T_p^2 + T_{sky}^2 \right) \cdot \left(T_p - T_f \right) + \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_{sky} \right) \cdot \left(T_p^2 + T_{sky}^2 \right) \cdot \left(T_f - T_{sky} \right) \qquad (17)$$

with

$$U_{ps} := \varepsilon_{eff} \cdot \sigma \cdot \left(T_p + T_{sky} \right) \cdot \left(T_p^2 + T_{sky}^2 \right) \qquad (18)$$

and hence

$$\dot{q}_{sky} = U_{ps} \cdot \left[\left(T_p - T_f \right) + \left(T_f - T_{sky} \right) \right] \qquad (19)$$

Here, the term with the forth power is eliminated completely which leads to a lower sensitivity regarding measured data of the infrared irradiance. Furthermore, ε_{eff} is integrated in U_{ps} so that this linearization leads to a lower number of model parameters.
After eliminating (T_p-T_f) and rearranging eq.(1), described in detail in (Vajen et al. 2003), the model equation for the power delivery of the collector can be evaluataed. For the approach of (EN 12795, 2002) one receives for example

$$\begin{aligned} \dot{q}_{use} = & \left(\frac{h_i}{h_i + U_{pa} + U_{c0} + U_{c1} \cdot v_w} \right) \cdot \alpha \cdot G \\ & - \left(\frac{h_i}{h_i + U_{pa} + U_{c0} + U_{c1} \cdot v_w} \right) \cdot \left(U_{pa} + U_{c0} + U_{c1} \cdot v_w \right) \cdot \left(T_f - T_a \right) \\ & - \left(\frac{h_i}{h_i + U_{pa} + U_{c0} + U_{c1} \cdot v_w} \right) \cdot \varepsilon_{eff} \cdot \sigma \cdot \left(T_a^4 - T_{sky}^4 \right) \\ & + U_b \cdot \left(T_f - T_a \right) \\ & - C_{eff} \cdot \frac{\partial T_f}{\partial t} \end{aligned} \qquad (20)$$

In this equation one faces an aggregation of parameters (h_i, U_x) and boundary conditions (v_w). Therefore, another approximation has to be carried out: A factor F^* is introduced which accumulates several parameters, depending on the different mathematical transformations described above. F^* is somehow equivalent to the well known collector-efficiency factor F', but in contrary to F', F^* does not include the heat losses at the rear side of the absorber.
In the model equation a transformation is made to separate the terms depending on the wind speed. The factor F^* can now be approximated as follows:

$$F_{pa}{}^* = \frac{h_i}{h_i + U_{pa} + U_{c0} + U_{c1} \cdot v_w} \approx \frac{h_i - U_{pa} - U_{c0} - U_{c1} \cdot v_w}{h_i} \qquad (21)$$

Assuming

- a wind speed of 2 m/s,
- h_i= 200 W/(m²K),
- U_{c0}= 2 W/(m²K) and
- U_{pa} = 3 W/(m²K)

this approximation leads to a relative deviation of about 1%. However, the accuracy of the approximation is a function of the wind speed and it can not be assumed in general that the precondition $U_{c1} \cdot v_w << h_i$ is valid.
Fig. 1 illustrates the relative deviation between the two terms in eq. (21). Thus, a seasonal average deviation of 1 to 3% due to the approximation has to be expected. For different locations with differing wind speed distributions the deviation may be significantly smaller or higher.

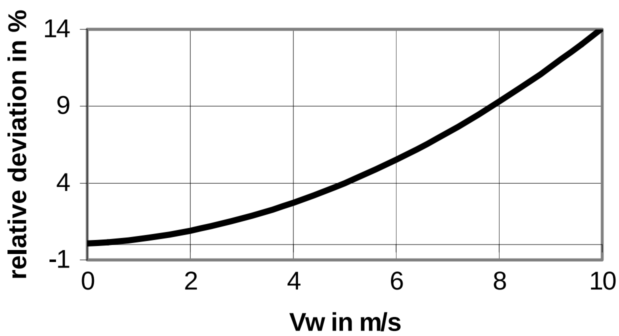

Fig. 1: Relative deviation of the terms of eq. (21) as a function of the wind speed. Parameters: h_i = 200 W/(m²K), U_{pf} =3 W/(m²K), U_{c0} = 2 W/(m²K), U_{c1} = 7 Ws/(m³K)

For the other models (Vajen et al. 1993 and 2003) the error due to the approximation in eq. (21) is similar to the one above. The final equation for the useful energy gain of the model (EN 12795, 2002) results to:

$$
\begin{aligned}
\dot{q}_{use} = {} & \left(\frac{h_i - U_{c0} - U_{pa}}{h_i}\right)\cdot\alpha\cdot && G \\
& -\frac{U_{c1}}{h_i}\cdot\alpha\cdot && v_w \cdot G \\
& -\left\{\left[\frac{h_i - U_{c0} - U_{pa}}{h_i}\right]\left(U_{c0} + U_{pa}\right) + U_b\right\}\cdot && \left(T_f - T_a\right) \\
& -\left(\frac{h_i - 2U_{c0} - 2U_{pa}}{h_i}\right)\cdot U_{c1}\cdot && v_w \cdot \left(T_f - T_a\right) \\
& +\frac{U_{c1}^2}{h_i}\cdot && v_w^2 \cdot \left(T_f - T_a\right) \\
& -\varepsilon_{eff}\cdot\sigma\cdot\left(\frac{h_i - U_{c0} - U_{pa}}{h_i}\right)\cdot && \left(T_a^4 - T_{sky}^4\right) \\
& +\varepsilon_{eff}\cdot\sigma\cdot\frac{U_{c1}}{h_i}\cdot && v_w \cdot \left(T_a^4 - T_{sky}^4\right) \\
& -C_{eff}\cdot && \frac{\partial T_f}{\partial t}
\end{aligned}
\qquad (22)
$$

In (EN 12975, 2002) the parameters are bundled together to constants c_1 to c_6 which can not be interpreted physically any more. Furthermore, a term equivalent to $v_w^2 \cdot (T_f - T_a)$ is missing, whereas a term equivalent to $(T_f - T_a)^2$ appears without physical reason. Additional errors occur due to the different approximations of U_x describing the radiant heat exchange in eq. (14) and (19) which affects the accuracy of the calculated useful system power. For T_p=280 K and $T_{sky,\ ref}$ = 290 K, a comparison of eq. (19) and (5) by eq. (23)

$$
err_{rel,sky} = 100\cdot\frac{\varepsilon\cdot\sigma\cdot\left(T_p^4 - T_{sky}^4\right) - U_{ps}\cdot\left(T_p - T_{sky}\right)}{\varepsilon\cdot\sigma\cdot\left(T_p^4 - T_{sky}^4\right)} \qquad (23)
$$

delivers a deviation of more than 0,5 % / K. Thus, already a variation of T_{sky} by 5 K leads to an error in the model of about 2,5%. Regarding the other model equations (14) and (16) the deviation is similar.

Sensitivity analysis of the behaviour of the models

To get an impression of the behaviour of the different models, a sensitivity analysis has been carried out regarding varying boundary conditions. For this purpose, reference values of boundary conditions and parameters have been defined (Tab. 1). Further, T_{sky}, T_a, T_i and G have been varied. The deviations have been calculated with respect to the physical model, eq. (1), which has been solved iteratively and serves as a reference model. The results are plotted in the following figures. Here, "deviation" always means the relative error of the useful power gain of the linearized model equations, compared with the value calculated by the reference model.

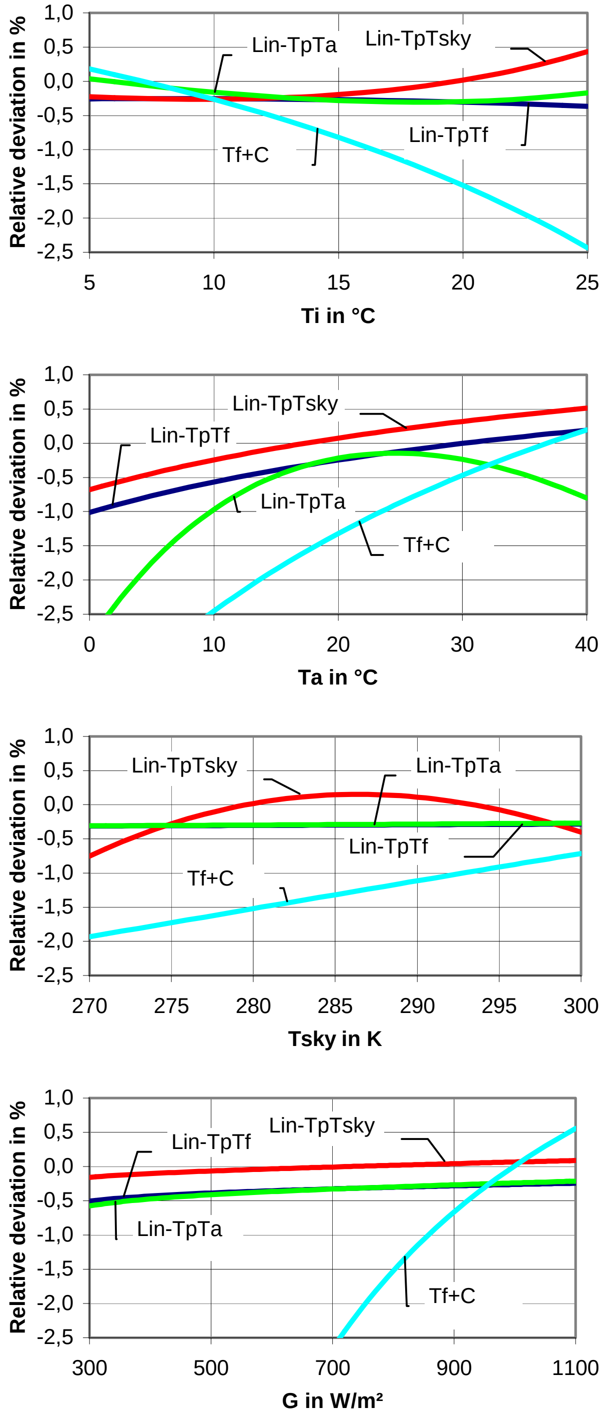

Fig. 2: Deviation of $\dot{q}_{use}$ for the different models, compared with the iteratively solved physical model, eq. (1) Lin-TpTa: eq. (12), Lin-TpTf: eq.(15), Lin-TpTsky: eq. (18), Tf+C: eq. (9)

Tab. 1: Boundary conditions for the model tests

T_i	293,15 K (20°C)
G	800 W/m²
T_a	291,15 K (18°C)
T_{sky}	280,0 K (6,85°C)
v_w	2 m/s
ε_{eff}	0,8
$\alpha_\perp$	0,95
h_i	200 W/(m²K)
C_{eff}	4,6 Wh/(kgK)
U_{c0}	1,0 W/(m²K)
U_{c1}	3,0 Ws/(m³K)
$\dot{v}$	4 m³/h

As shown in fig. 2, the simplified model equations tend to underestimate the power prediction of the reference model. This is not necessarily the case if the parameters of the absorber have been fitted instead of being defined.
Furthermore, obviously model eq. (9), $T_p=T_f+C$, is not suitable. A variation of T_i or G does not seem to be critical for the three remaining models.
Major deviations occur for the model containing the varied boundary condition in the linearized term.
As expected, the maximal deviations occur if linearization is carried out for boundary conditions. Especially for the model Lin-TpTa, eq. (13) the highest deviations appear for different ambient temperatures. Therefore, based on these investigations, the models Lin-TpTsky, eq. (19), (Perers 1987), or Lin-TpTf, eq. (16), (Vajen et al. 2003), seem to be most suitable.
Simply varying only one factor (e.g. T_{sky} or G) doesn't allow an adequate final conclusion. It has to be investigated, how model parameters (e.g. U_x) can be fitted to measured data and how the system behaves under changing boundary conditions.

Conclusions and Outlook

The models of (Perers 1987) and (Vajen et al. 2003) delivered the most accurate predictions of the useful power gain for uncovered solar collectors, whereas the equation based on (EN 12975, 2002) showed major deviations, especially for varying ambient temperatures. The "pure" equation in the European norm could not be included in the comparison, because it contains terms which can not be derived from the energy balance, eq. (1). The EN-equation might be proper for covered collectors with $T_{cover} \cong T_a$, but it seems to be less suitable for uncovered collectors.

However, a general statement regarding the quality of different models cannot be delivered yet. The investigations and comparisons carried out were based on physically reasonable heat transfer parameters in the model equations. A comparison with parameters fitted to measured values might lead to different results.
The linearized parameters U_x implicitly contain material properties and boundary conditions, i.e. the weather conditions during the collector tests. Thus, further investigations are necessary concerning the sensitivities of different parameters on the model's result and the possibility to split U_x in material properties and boundary conditions. Compared to the other models taken into account, the approach of (Perers 1987) contains one parameter less. However, this parameter reduction could be achieved as well with a different formulation of (Vajen et al. 2003), if U_{pf} is split into $\varepsilon\cdot\sigma$ and $(T_p+T_f)(T_p^2+T_f^2)$, because T_p and T_f are nearly constant in the usual range of operating conditions of uncovered collectors. Nevertheless, it can be recommended to carry out an iterative determination of T_p on the basis of eq. (1).

Nomenclature

c_p	Wh/(kgK)	specific heat capacity
h_i	W/(m²K)	inner heat transfer coefficient
$\dot{q}_b$	W/m²	backside heat transfer
$\dot{q}_{conv}$	W/m²	convective heat losses
$\dot{q}_{sky}$	W/m²	radiant heat losses
$\dot{q}_{use}$	W/m²	useful power gain
T_a	K	ambient temperature
T_f	K	average fluid temperature
T_i	K	fluid inlet-temperature
T_o	K	fluid outlet-temperature
T_p	K	average surface-temperature of the absorber
T_{sky}	K	sky temperature
U_b	W/(m²K)	back heat transfer coefficient
U_c , U_{c0}	W/(m²K)	convective heat transfer
U_{c1}	Ws/(m³K)	coefficients
U_{pa} , U_{ps}	W/(m²K)	radiant heat transfer coefficients

v_w	m/s	average wind speed
α		Absorption coefficient
ε_{eff}		effective emission coefficient
ref		reference

Acknowledgements

The authors like to thank especially the following institutions for financial and logistical support: Volkswagen-Stiftung which supports this research, The Center of the Problems of Unconventional and Renewable Resources of Energy in Bishkek (Kyrgyzstan), Kyrgyz Technical University, and the company Bishkek Teploenergo.

References

(EN 12975, 2002): **EN12975-2:2002 (2002)**: Thermal solar systems and components, Part 2: Test methods

(ISO 9806, 1995): **ISO 9806-3 (1995)**: Test methods for solar collectors – Part 3: Thermal performance of unglazed liquid heating collectors (sensible heat transfer only) including pressure drop

(Perers 1987): **Perers, B. (1987)**: Performance Testing of unglazed collectors, Report for IEA Task III, Studsvik Energy, Sweden

(Rockendorf et al 2001): **Rockendorf, G.; Sillmann, R.; Bethe, T.; Köln, H. (2001)**: Solare Freibadbeheizung, Absorberprüfung und Testergebnisse Anlagen – Planung und Betrieb, Institut für Solarenergieforschung, Hameln / Emmerthal

(Vajen et al. 1999): **Vajen, K. ; Krämer, M. ; Orths, R.; Boronbaev, E.K. (1999)**: Solar Absorber System for Preheating Feeding Water for District Heating Nets, Proc. ISES Solar World Congress, 4.-9.7 1999, Jerusalem, Vol. III, pp. 90-91

(Vajen et al. 2003): **Vajen, K. ; Frank, E. ; Krämer, M.** : Extended Model Equations for Uncovered Collectors, ISES Solar World Congress, Gothenburg, 2003